Physics 151 Class Exercise: Projectile Motion

1. A golfer tees off on level ground, giving the ball an initial speed of 52 m/s and in initial direction of 32° above the horizontal.

   a) Make a drawing of the golfer and the ball’s trajectory. Clearly indicate the origin and the direction of the x and y axes.

   ![Graph of a parabolic trajectory with x and y axes labeled](image)

   b) Calculate the initial velocities in the x and y directions.

   \[
   v_{ox} = v_0 \cos \theta = \left( \frac{52 \text{ m}}{s} \right) \cos(32°) = 44.1 \frac{\text{m}}{s} \\
   v_{oy} = v_0 \sin \theta = \left( \frac{52 \text{ m}}{s} \right) \sin(32°) = 27.6 \frac{\text{m}}{s}
   \]

   c) Calculate the length of time it takes the ball to reach the peak of its trajectory?

   Known: 
   
   \[
   v_{oy} = 27.6 \frac{\text{m}}{s} \quad t \\
   v_y = 0 \\
   a = -9.81 \frac{\text{m}}{s^2}
   \]

   Solve: 
   
   \[
   v_y = v_{oy} + at \\
   t = \frac{v_y - v_{oy}}{a} = \frac{-27.6 \frac{\text{m}}{s}}{-9.81 \frac{\text{m}}{s^2}} = 2.81 \text{s}
   \]

   Not Involved: 
   
   y

   d) Calculate the total length of time the ball is in the air. Total time = 2 * Peak time = 5.62 s

   e) Calculate the distance from the tee where the ball lands. 
   
   \[
   x = v_x t = (44.1 \frac{\text{m}}{s})(5.62 \text{s}) = 248 \text{m}
   \]

   f) Check this value by recalculating it using the Horizontal Range formula.

   \[
   R = \frac{v_0^2}{g} \sin 2\theta = \left( \frac{52 \frac{\text{m}}{s}}{9.81 \frac{\text{m}}{s^2}} \right)^2 \sin 64° = 248 \text{m}
   \]
2. An artillery officer is practicing on a firing range on a flat stretch of ground. She endeavors to hit a target 885m away with an artillery shell. The artillery gun fires shells with a muzzle velocity of 96.1 m/s.

a) At what angles can she orient the gun. (Hint: Consider the angles/quadrants where two angles have the same sin value.)

\[
\sin 2\theta = \frac{gR}{v_k^2} = \frac{\left(9.81 \frac{m}{s^2}\right)(885m)}{\left(96.1 \frac{m}{s}\right)^2} = 0.94
\]

Thus \(2\theta\) can be 70\(^\circ\) or 110\(^\circ\). Thus \(\theta\) can be 35\(^\circ\) or 55\(^\circ\)

b) What is the difference in “time to impact” for the two trajectories.

\[
t_1 = \frac{v_{oy}}{a} = \frac{\left(96.1 \frac{m}{s}\right)\sin 35^\circ}{\left(9.81 \frac{m}{s^2}\right)} = 5.6s
\]

\[
t_2 = \frac{v_{oy}}{a} = \frac{\left(96.1 \frac{m}{s}\right)\sin 55^\circ}{\left(9.81 \frac{m}{s^2}\right)} = 8.0s
\]

\(\Delta t = 2.4s\)

Since this is the difference in time to the peak – the total difference for the path is 4.8s.

c) What is the difference in “peak height” for the two trajectories.

Known: Solve: Not Involved:

\[v_{oy} = 55.1 \text{ m/s}\]

\[v_y = 0\]

\[a = -9.81 \text{ m/s}\]

\[v_y^2 = v_{oy}^2 + 2ay\]

\[
y_1 = \frac{-v_{oy}^2}{2a} = \frac{\left(96.1 \frac{m}{s}\sin 35^\circ\right)^2}{2\left(9.81 \frac{m}{s^2}\right)} = 154.9m
\]

\[
y_1 = \frac{-v_{oy}^2}{2a} = \frac{\left(96.1 \frac{m}{s}\sin 55^\circ\right)^2}{2\left(9.81 \frac{m}{s^2}\right)} = 315.8m
\]

\(\Delta y = 161m\)

d) Draw a crude sketch of the gun, target, and the two trajectories in the space below.