Physics 151 Class Exercise: Simple Harmonic Motion

1. A 1.4 kg mass is attached to a large horizontal spring on the top of a table. The mass is pulled 12 cm from the equilibrium position and released. It then undergoes simple harmonic motion making 2.2 oscillations each second. Determine:

(a) the equation of motion

\[ x(t) = x_0 \cos \left( \frac{2\pi t}{T} \right) = (0.12m) \cos (13.8t) \]

(b) the spring constant

\[ f = \frac{1}{2\pi} \sqrt{\frac{k}{m}} \]

\[ f^2 = \frac{1}{4\pi^2} \frac{k}{m} \]

\[ k = 4\pi^2 f^2 m = 4\pi^2 (2.2Hz)^2 (1.4kg) = 267.5 \frac{N}{m} = 267 \frac{N}{m} \]

c) the total energy

\[ E = \frac{1}{2} k x_0^2 = \frac{1}{2} \left( \frac{267.5 \frac{N}{m}}{m} \right) (0.12m)^2 = 1.926J = 1.93J \]

d) the maximum acceleration of the mass (and indicate where this occurs)

\[ a = \frac{kx_0}{m} = \frac{\left( \frac{267.5 \frac{N}{m}}{m} \right)(-0.12m)}{(1.4kg)} = 22.9 \frac{m}{s^2} \text{ the negative amplitude position} \]

e) the maximum velocity of the mass (and indicate where this occurs)

\[ v = x_0 \sqrt{\frac{k}{m}} = (0.12m) \sqrt{\left( \frac{267.5 \frac{N}{m}}{m} \right) \left( \frac{1}{(1.4kg)} \right)} = 1.66 \frac{m}{s} \text{ at the equilibrium position} \]

f) the acceleration of the mass when it is 7 cm from the equilibrium position

\[ a = -\frac{kx}{m} = -\frac{\left( \frac{267.5 \frac{N}{m}}{m} \right)(0.07m)}{(1.4kg)} = -13.4 \frac{m}{s^2} \]

g) the velocity of mass when it is 7 cm from the equilibrium position

\[ E = \frac{1}{2} mv^2 + \frac{1}{2} kx^2 \]

\[ v = \sqrt{\frac{2E - kx^2}{m}} = \sqrt{\frac{2(1.93J) - \left( \frac{267.5 \frac{N}{m}}{m} \right)(0.07m)^2}{(1.4kg)}} = 1.35 \frac{m}{s} \]