Physics 151 Class Exercise: Vectors2 - KEY

1. A basketball player runs down the court, following the path indicated by the vectors A, B, and C in the figure below. The magnitudes of these three vectors are A = 10.0 m, B = 20.0 m, and C = 7.0 m. Find the magnitude and direction of the net displacement of the player.

   ![Vector diagram]

a) Estimate the solution using the graphical method.

   Displacement ~ 20 m
   Angle ~ 0º

b) Calculate an accurate solution using the analytical component method. Does your answer agree with the result of part a?

   Define a coordinate system where the x-axis increases to the right and the y-axis is toward the top of the page.

<table>
<thead>
<tr>
<th>Vector</th>
<th>x-coord</th>
<th>y-coord</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0</td>
<td>-10.0 m</td>
</tr>
<tr>
<td>B</td>
<td>(20.0 m) cos (45º)</td>
<td>(20.0 m) sin (45º)</td>
</tr>
<tr>
<td>C</td>
<td>(7.0 m) cos (30º)</td>
<td>-(7.0 m) sin (30º)</td>
</tr>
<tr>
<td>R</td>
<td>20 m</td>
<td>0.60 m</td>
</tr>
</tbody>
</table>

   \[
   \vec{A} + \vec{B} + \vec{C} = \sqrt{(20 \text{ m})^2 + (0.60 \text{ m})^2} = \frac{20 \text{ m}}{20 \text{ m}} = 1.7º
   \]
2. The pilot of an airplane wishes to fly due north, but there is a 75 km/h wind blowing toward the east. (a) In what direction should the pilot head her plane if its speed relative to the air is 310 km/h?

**Solution from Text**

\[ \vec{v}_{pg} = \text{velocity of the plane relative to the ground} \]

\[ \vec{v}_{pa} = \text{velocity of the plane relative to the air} \]

\[ \vec{v}_{ag} = \text{velocity of the air relative to the ground} \]

\[ \vec{v}_{pg} = \vec{v}_{pa} + \vec{v}_{ag} \]

The plane needs to travel due north. Let north be along the positive \( y \)-axis, then

\[
\vec{v}_{pg} = \left( 310 \frac{\text{km}}{\text{h}} \right) (\cos \theta) \hat{x} + \left( 310 \frac{\text{km}}{\text{h}} \right) (\sin \theta) \hat{y} + \left( 75 \frac{\text{km}}{\text{h}} \right) \hat{x}
\]

For the plane to travel due north, the net velocity in the \( x \)-direction relative to the ground must equal zero.

\[
v_{pgx} = 0 = \left( 310 \frac{\text{km}}{\text{h}} \right) \cos \theta + \left( 75 \frac{\text{km}}{\text{h}} \right)
\]

\[ \cos \theta = -\frac{75 \frac{\text{km}}{\text{h}}}{310 \frac{\text{km}}{\text{h}}} = -\frac{15}{62} \]

\[ \theta = \cos^{-1} \left( \frac{15}{62} \right) = 104^\circ = 14^\circ \text{ west of north} \]

(b) Draw a vector diagram that illustrates your result in part (a).

(c) If the pilot decreases the air speed of the plane, should the angle found in part (a) be increased or decreased?

If the plane’s speed is decreased, the angle should be increased.

**Instructor’s Solution**

Occasionally the textbook’s usage of the unit vector notation really overcomplicates matters.

\[ \theta = \tan^{-1} \left( \frac{75}{310} \right) = 14^\circ \]

Angle specified west of north